# Grete Hermann, Quantum Mechanics, and the Evolution of Kantian Philosophy

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**Short biography of Grete Hermann (1901–1984)**

Grete Hermann was born in Bremen, in the northwestern corner of the then German Empire, to Augusta and Gerhard Hermann on 2 March 1901. She obtained her doctoral degree in mathematics, with minors in physics and philosophy, at Göttingen in 1925 under Emmy Noether, the first (and as it turned out the only female) doctoral student that Noether was to officially supervise. Hermann's teacher in philosophy was Leonard Nelson, a neo-Kantian philosopher of the neo-Friesian school. She pursued her philosophical interests after obtaining her doctorate, and became Nelson's private assistant, working with him until his death in 1927 on his volume on philosophical ethics and pedagogy, which she published posthumously with Minna Specht in 1932. After Nelson's death she became a leading member of the International Socialist Combat League, which was suppressed by the Nazis in 1933. She remained in Germany during the first few years of the Nazi period, travelling occasionally to Denmark to visit the exile schools operated there by Specht. It was at this time that Hermann became acquainted with Werner Heisenberg and his then assistant, Carl Friedrich von Weizsäcker. She became deeply interested in claims by Heisenberg and other physicists that quantum mechanics had invalidated Kant's causal law. She continued her conversations with Bohr, Heisenberg, von Weizsäcker and others and in 1935 Hermann published her reflections on quantum theory's significance for Kantian philosophy. Her paper was glowingly reviewed by von Weizsäcker a year later in *Physikalische Zeitschrift*. Heisenberg would later devote an entire chapter of his scientific autobiography to his discussions with Hermann, remarking on the mutually beneficial nature of their interactions. She emigrated to Denmark in 1934, and then to London around the turn of 1937–8, where she married Edward Henry. They remained married for twenty-two years. Returning to Germany after the war, she was eventually appointed full professor at the Pädagogische Hochscule in Bremen in 1950. Although she maintained contact with the physics community, in her later career she focused her energies primarily in the fields of ethics and pedagogy, publishing a long commentary on Nelson's work in 1953. Hermann retired in 1966 but remained active academically until late in life. She died on 15 April 1984 in Bremen.

## A Introduction

Even before Hegel's death in 1831, the idea, traditional since the ancient Greeks, that philosophical speculation could furnish a general model for scientific thought had been waning in the Germanic world. What were seen as the excesses of the romantic and idealist movements, combined with the rapid growth and manifest success of empirical science, had brought philosophy into disrepute. Some—the so-called 'vulgar materialists'—went so far as to call for philosophy to essentially identify itself with empirical science (Schnädelbach 1984, ch. 3; Beiser 2014, pp. 182-188).

The neo-Kantian movement of the mid-nineteenth century—itself a revival of an earlier trend of thought contemporaneous with speculative idealism and exemplified by thinkers such as Jakob Friedrich Fries (Beiser, 2014, ch. 1)—arose partly as a reaction to these intellectual currents. Championed by natural scientists such as Hermann von Helmholtz (Schnädelbach, 1984, p. 103), the movement did not, as did speculative idealism, view it as philosophy's task to produce scientific knowledge by pure thought alone. Nor did these neo-Kantians, with the vulgar materialists, consider it philosophy's task merely to uncritically catalogue and systematise the results of empirical science. Rather, with Kant, they saw it as philosophy's essential, unavoidable, and enduring mission to enquire into the sources of our knowledge and the degree of its justification.[1] In other words they, and the philosophers such as Liebmann, Lange, and Cohen who took up their battle call (Zeman, 1997), saw the proper task of philosophy as consisting in the provision of an *epistemological foundation* for science. By the close of the nineteenth century, neo-Kantianism exerted a powerful influence on Germanic thought.

It was against this intellectual backdrop that many spectacular achievements in logic, mathematics, and physical science were made. Some of these were to eventually deal a heavy blow to the popularity of Kant's philosophy. Pure logic, Kant had argued, could never provide us with a genuine expansion of our knowledge; yet Frege's *Begriffsschrift* and the later systems that drew from it seemed to provide us with tools to do just this. Euclidean geometry was held

[1]... sondern sie beabsichtigte nur, die Quellen unseres Wissens und den Grad seiner Berechtigung zu untersuchen, ein Geschäft, welches immer der Philosophie verbleiben wird, und dem sich kein Zeitalter ungestraft wird entziehen können (Helmholtz, 1855, p. 5).

by Kant to be both a synthetic and an a priori science. Yet Hilbert was able to show that it followed analytically from a set of basic axioms, and the development and physical application (in relativity theory) of non-Euclidean geometry showed that no one particular geometry could be regarded as a priori true. Arguably worst of all for Kant, the development of quantum theory seemed to tell against according a fundamental physical status to the principle of cause and effect. By the middle of the twentieth century it was widely held that Kant's philosophy had been definitively refuted.[2]

Yet the truth is more subtle than this. Many of the late-nineteenth and early twentieth century thinkers whose work had contributed to the demise in popularity of Kantian philosophy were, despite their divergences from him, substantially influenced by Kant. Frege, for example, is at pains to call attention to his agreement with Kant, which he claims far exceeds the extent of his disagreement (Frege, 1980 [1884], §89).[3] In the epigraph to his seminal work on geometry, Hilbert expresses the affinity of his thought with Kant's by invoking the latter in support of the spirit of his investigations (Hilbert, 1902, p. 1).[4] Reichenbach's conception of the relativised a priori, the conventionalisms of Poincaré, Schlick, and Carnap, the pragmatisms of C. I. Lewis and others, are best characterised, not as radical rejections of Kant's philosophical framework, but as attempts to re-explicate the basic Kantian idea that any framework of theoretical knowledge must include a part—what Kant had (mistakenly, according to these thinkers) called the synthetic a priori—that is conceptually and epistemologically distinct from the rest of that framework. Viewed as a research program (cf. Bitbol, 2017), one may say that Kant's transcendental approach to philosophy continued, and continued to evolve, albeit along multiple independent pathways, well into the last century. It is only with the rise of the Quinean holistic conception of science that these ideas are rejected in their totality.[5]

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[2]See, for instance, Carnap's summary of the prevailing attitude toward Kant in Reichenbach (1958, p. vi).

[3]For more on the parallels between Kant and Frege, see Cuffaro (2012); Merrick (2006).

[4]For more on the Kantian aspects of Hilbert's thought, see Kitcher (1976).

[5]I am of course not claiming that all (or even any) of the thinkers mentioned in this paragraph would have called themselves Kantians; in particular Schlick and (the later) Carnap certainly would not have done so, and nor should we. I nevertheless do think that their views on these matters as well as the views of the others I have mentioned can be seen as continuous with (in the sense of evolving continuously out of) a research programme that was begun by Kant. For more on these topics, see Coffa (1991); DiSalle (2002); Friedman (2009); Howard

The development of quantum theory in the early part of the last century posed a particularly strong challenge to the Kantian philosophical viewpoint. A common opinion expressed at the time was that the indeterminacy intrinsic to the theory definitively refutes Kant's philosophy insofar as it shows that Kant was wrong to ascribe a priori status to the principle of cause and effect. As with other developments in the mathematical and natural sciences during the period, however, the relationship between quantum theory and Kant's theoretical philosophy is far richer and more interesting than this, and there has been a flowering of scholarship in recent years exploring the relationship between Kantian philosophy and quantum mechanics, especially with regard to the views of the influential physicist Niels Bohr (see, e.g., Bächtold, 2017; Bitbol, 2017; Chevalley, 1994; Cuffaro, 2010; Kaiser, 1992; Kauark-Leite, 2017; Pringe, 2009).

At the very heart of Kant's critical philosophy is the question concerning the constraints on and preconditions for objective cognition—the general conditions under which we can be said to know something objectively. This question is arguably more pressing in quantum mechanics than in any other area of physics (see Pitowsky, 1994, sec. 6). And it is precisely the question that occupied physicists, such as Bohr, in their efforts to interpret quantum mechanics in the initial stages of the theory's development. It is also the question that occupied Grete Hermann, a neo-Kantian philosopher-mathematician who productively and mutually beneficially interacted with Bohr and others of the founders of quantum mechanics during the theory's early period. Hermann was a student of Emmy Noether's who had in addition studied with the neo-Friesian philosopher Leonard Nelson, and she considered herself a Neo-Friesian as well. Fries, like Kant, had understood critical philosophy to be an essentially epistemic project. Fries departed from Kant, however, in his account of the elements involved in our cognition. Below we will see how, beginning from a neo-Friesian understanding of critical philosophy, Hermann is led to conclude that quantum mechanics shows us that physical knowledge is fundamentally 'split': that the objects of quantum mechanics are only objects from a particular perspective and in the context of a particular physical interaction. We will see, further, how Hermann's particular solution to the problem of objectivity in quantum mechanics is at odds with those offered by

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(1994); Murphey (2005).

more orthodox versions of Kantian doctrine even though it is natural from a neo-Friesian point of view.[6]

More concretely: we will begin, in Section B, by introducing the core of the intellectual tradition that Hermann took herself to be a part of: Kant's doctrine of transcendental idealism. We will see that it is an epistemic doctrine but that it rests on an ontological posit of a sort: the discursivity thesis, a thesis Kant grappled with in his struggle to provide the first principles for metaphysical cognition in the period leading up to and including the publication of his inaugural dissertation. We will see in Section C how Kant's search for the first principles of metaphysical cognition is transformed, during the critical period, into the search for the first principles of synthetic a priori cognition, and in Sections D–E we will discuss the challenges faced by Kant's view which arise from the emergence of quantum theory. In Sections F–J we will discuss Grete Hermann's assessment of these challenges to Kantian critical philosophy. In Section F we will see how Hermann rejects the call to supplement quantum mechanics with additional variables, rather affirming that quantum mechanics provides the resources with which to identify the causes of the processes it describes. In Section G we will see how Hermann's solution to the problem of objectivity in quantum mechanics departs from orthodox Kantian doctrine. Hermann was a neo-Friesian neo-Kantian, and in Section H we will introduce the major elements of (neo-)Friesian thought, noting in particular the Friesian insistence that the fundamental principles of cognition be discovered through a process of reflection on the actual, situated, contents of cognition. We will see in Section I how Hermann is led to conclude that

[6]In her illuminating chapter on Hermann's philosophy, Elise Crull (2017) also understands Hermann's project as, primarily, one of laying out these broader consequences of reflecting on quantum mechanics for our theoretical knowledge, as opposed to (merely) an attempt to reconcile quantum mechanics with Kant's principle of causality (although, of course, reconciling quantum mechanics with causality is one part of Hermann's broader project). Part of the contribution of my chapter is to further advance this basic take, which I share with Crull, on Hermann's views, through a more in-depth conceptual investigation of Hermann's Kantian and Friesian roots as these relate to transcendental idealism in particular. One point on which I disagree with the position put forward in Crull (2017), however, is her ontological reading of Hermann's doctrine of the splitting-of-truth (p. 165). I do not have the space to discuss this issue further in this chapter, but it is discussed in depth in Cuffaro (2018, sec. 10), and also, much more briefly, in footnotes 7 and 30 below. Recently, I have learned through personal correspondence that Crull has since (independently) come to change her view on this aspect of Hermann's doctrine, and I believe we are now largely in agreement on the issue.

physical knowledge is fundamentally ‘split’, and how Hermann’s solution to the problem of objectivity in quantum mechanics is rooted in the neo-Friesian tradition of which she partook. In Section J we will argue that despite her departure from Kant, Hermann, and neo-Friesians generally, remain very much Kantian in spirit. Section K concludes the chapter.

## B Kant’s transcendental idealism

Kant’s transcendental idealism is an *epistemic* doctrine (Allison, 2004, ch. 1).[7] That is to say, on Kant’s view, implicit in the concept of objective cognition are the subjective—i.e., epistemic—conditions under which an object is representable to us. In particular, on Kant’s view our cognition is *discursive* in the sense that we possess both a sensible and a conceptual faculty; the former being that through which our intuitions of external objects are mediated, and the latter being the faculty through which we spontaneously act upon these intuitions in order to subsume them under general concepts. Discursivity amounts to the claim that we require *both* in order for cognition to be possible. As Kant famously puts it in the first Critique: “Thoughts without content are empty; intuitions without concepts are blind” (A51/B75).[8]

By abstracting completely away from the conditions pertaining to our sensibility, the idea of an object *regarded as it is in itself* can still be thought, but the idea so formed is not an object that we can know, for cognition requires (in virtue of the discursivity thesis) the contributions of both the faculties of sensibility and understanding. As regards things in themselves transcendental idealism is thus a *conceptual* doctrine: it asserts that a thing conceived of as it

[7]I am asserting, without argument, that the so-called ‘epistemic reading’ of transcendental idealism is the correct one. There is a fair deal of controversy on this point, however. Interpreters of Kant such as Guyer (1987) and Langton (1998), for instance, prefer a reading of transcendental idealism that is in some sense ontic (for a summary of the controversy, see Stang 2016, sec. 4.4). I do not have enough space to defend the epistemic reading here, even though it could be defended on the basis of what we will be discussing in the sequel (see Cuffaro, 2018). Neo-Friesians as a rule subscribed to the epistemic reading in any case (even if they held Kant himself to be confused regarding its true significance) and that justifies our adopting it here (see Fries 1828, pp. xxiv–xxv; Hermann 2017, p. 271; Nelson 1970, pp. 190–197; and Cuffaro 2018, sec. 10 for further discussion).

[8]References to the *Critique of Pure Reason*, or ‘first Critique’, will be to the Pluhar translation (1996). Page numbers for all of Kant’s works cited in this chapter (with the exception of the first Critique) will be as they are in the standard German edition. In the case of the first Critique, page numbers are as in the first (1781) and second (1787) edition, where “A” denotes the first and “B” the second, as usual.

is in itself cannot be an object of cognition for us, because in forming this concept of it we transcend the constraints, most generally given by the discursivity thesis, that any object knowable by us must satisfy. This does not imply that all talk of things in themselves should be thought of as empty and without significance (cf. Ameriks, 2003, pp. 28, 32, 56), nor that the concept of a thing regarded as it is in itself is either ontologically less (or more) privileged than the concept of a thing regarded under the conditions according to which it can be cognised by us (Allison, 2004, p. 47). What transcendental idealism proscribes is any inference that goes from sensible concepts to the nature of things in themselves, even when this inference concerns only their pure form (A26/B42).

This idea, expressed by the discursivity thesis, that there can be no standpoint-independent perspective from which to know (cf. Stang 2016, sec. 4.3.3), is what Allison (2004, ch. 2) calls the 'anthropocentric paradigm' of cognition. It contrasts with the older 'theocentric paradigm'. On this view, to know something is to know it absolutely independently of the way it is considered. Understood in this way, both transcendental idealism and the 'transcendental realism' associated with the theocentric paradigm express norms or standards by which to evaluate cognition. They are *metaphilosophical* doctrines (Allison, 2004, p. 35).[9]

Although transcendental idealism is an epistemic doctrine, the thesis on which it rests—discursivity—is an ontological posit of a kind, but it is one that relates to the nature of theoretical cognition rather than to the mind-independent world. Providing a principled grounding for theoretical cognition had long been a central concern of Kant's. The particular kind of theoretical cognition that concerned him as he entered the 1770s was, just as it had been in the 1750s and 1760s,[10] metaphysical cognition: cognition of things as they exist in themselves without regard to the forms through which we apprehend them; i.e., cognition as it is understood in the theocentric paradigm. As he puts it in his *Inaugural Dissertation* of 1770,

> whatever cognition is exempt from such subjective conditions relates only to the

[9]In the sequel I will (following Allison) be referring to transcendental idealism variously as an 'epistemic interpretation', 'conceptual interpretation', and 'metaphilosophical interpretation', in the senses in which those terms have been informally defined above. These descriptions of transcendental idealism are clearly not mutually exclusive.

[10]Clear examples of this can be found in: Kant (2002b, 1:387, 1:416), Kant (2002d, 2:66), and Kant (2002a, 2:285).

> object. It is thus clear that things which are thought sensitively are representations of things *as they appear*, while things which are intellectual are representations of things *as they are* (Kant, 2002c, 2:392).

He then argues that if we are to elucidate the form and principles of intellectual cognition (the proper concern of metaphysics) we must first identify and abstract from the form and principles of sensible cognition. As for the latter, Kant tells us “that these notions are not *rational* at all, and that they are not *objective* ideas of any connection, but that they are appearances, and that, while they do, indeed, bear witness to some common principle constituting a universal connection, they do not expose it to view” (Kant, 2002c, 2:391).

Rationally certain cognition with regard to the propositions of metaphysics is the goal which by the first Critique Kant will ultimately reject as unachievable. Yet in 1770 Kant is still hopeful that progress will be made in metaphysics once its methods have been purified (cf. also Kant, 2002a, 2:285). The aim of metaphysics as it was understood by Kant had always been rational certainty, but its methodology had always been ill-defined, and it had for the most part been hypothetical in character. By providing metaphysics with an epistemic grounding and a principled methodology it could be transformed into a science. He proposes two principles by which to do so. One of these is a kind of generalised principle of causality (Kant, 2002c, 2:407), or what Kant elsewhere had called the principle of the determining ground (see Kant, 2002b, §2, Proposition IV, 1:391-393). The other is the “principle of reduction” (Kant, 2002c, 2:413), by which any concept that refers in any way to sensibility—even only to its pure form—is excluded from the domain of objectively valid concepts (Kant, 2002c, 2:394).

By the time of the first Critique, Kant had become convinced that cognition without reference to the forms of our sensibility is impossible (cf. A5/B9). For even the principle of causality, the principle upon which he had grounded his hopes for such cognition (Kant 2002d, 2:79-80; Kant 2002c, 2:409), cannot survive the scrutiny of the principle of reduction. For the Kant of the first Critique, causality becomes an objectively valid principle only as it relates to appearances, the pure form of which is abstracted from sensibility. Nothing is left for metaphysical cognition. Cognition of the intelligible world, of things as they are independently of *all* conditions for the (possible or actual) experience of them, must be given up.

## C The form and principles of sensible cognition

By his own account, Kant had been awoken from his "dogmatic slumbers" with regard to causality through reading Hume (Kant, 2001, 4:260). For Kant it was clear, however, that the causal principle had been undermined only in regard to metaphysical cognition. Its use had not been undermined in the domain of natural science. He writes, in 1783:

> The question was not whether the concept of cause was right, useful, and even indispensable for our knowledge of nature, for this Hume had never doubted; but whether that concept could be thought by reason *a priori*, and consequently whether it possessed an inner truth, independent of all experience (Kant, 2001, 4:258–259).

He continues: "This was Hume's problem. It was a question concerning the *origin* of the concept, not concerning its indispensability in use. Were the former decided, the conditions of its use and the sphere of its valid application would have been determined as a matter of course" (ibid.).

Synthetic a priori cognition (a term Kant introduces in the first Critique) is that cognition in which two (or more) concepts are cognised, in advance of experience, as necessarily connected in some way with one another. Ampliative cognition of this kind is what metaphysics seeks. But by the 1780s Kant had concluded that synthetic a priori cognition in the sense required by metaphysics—cognition of things regarded independently of all conditions for the possible or actual experience of them—is impossible.[11]

This does not imply that all synthetic a priori cognition is impossible, however, and

[11]Near the beginning of Kant's so-called 'silent decade' in the 1770s (the period during which he produced the first Critique), he wrote this to Marcus Herz:

> [Lambert objects that changes] are possible only on the assumption of time; therefore time is something real ... Then I asked myself: Why does one not accept the following parallel argument? Bodies are real (according to the testimony of outer sense). Now, bodies are possible only under the condition of space; therefore space is something objective and real that inheres in the things themselves. The reason lies in the fact that it is obvious, in regard to outer things, that one cannot infer the reality of the object from the reality of the representation ... (Zweig, 1967, p. 75).

Kant's project now becomes to show *how* it is possible and to investigate its limits. These limits are *possible* experience, and since the *form* of possible experience is given a priori, synthetic a priori knowledge is possible regarding it. A useful *corollary* to the results of this investigation is a grounding for the theoretical sciences; i.e., an answer to the issue that Hume *did not* intend to raise: that of the validity and scope of principles, such as that of causality, *within* the theoretical sciences.[12] Metaphysics, regarded as a system of synthetic a priori constitutive principles which *transcend* experience, is declared impossible. Theoretical metaphysics in the traditional sense is transformed from a source of theoretical cognition into a system of methodological principles for the investigation of nature.[13]

The 'useful corollary' just mentioned takes up the first (smaller) part of his first Critique: the so-called *Transcendental Analytic*. Here, Kant explains that our experience of an object is comprised of two distinct aspects: *intuition*—the 'this', the 'that' of experience—and the *concepts* whereby one synthesises the manifold of intuition. Concepts belong to the faculty of *understanding*. Intuition is mediated by the faculty of *sensibility*: our mind's capacity to be affected by objects (A19/B33). The effect on sensibility of some object is called the *sensation* of it, and with sensation we associate the *empirical* aspect of our intuition.

Kant writes: "The undetermined object of an empirical intuition is called *appearance*" (A20/B34). A shape against the far wall in a dark room, for instance, which only after some scrutiny is determined to be a chair, is before this determination merely the appearance of something indeterminate.[14] There are two aspects to an appearance: its *matter*, i.e., *what* we sense in it, and the *form* through which the manifold corresponding to an appearance can be ordered. Appearance has two characteristic forms: space, associated with outer appearances, and time, associated with both inner and outer appearances. As forms of appearances, they are the *formal conditions* for appearances, in virtue of which they are known a priori as necessary relations according to which sensations must be ordered for subjects like us (A20/B34,

[12]Kant writes, at B20: "In solving the above problem we solve at the same time another one, concerning the possibility of the pure use of reason in establishing and carrying out all sciences that contain theoretical a priori cognition of objects." Cf. also: Kant (2001, 4:280).

[13]Our focus in this chapter remains theoretical *cognition*, so we will not be discussing (except incidentally) the role of such methodological principles here. It is easy to find such discussions in the literature. See, for instance, McLaughlin (1990).

[14]For a more thorough discussion of this point, see Harper (1984, p. 110-111).

A26/B42, A33/B49–50). They are also called 'pure' in virtue of not in themselves containing anything belonging to the matter of sensation (A20/B34).

Concepts of the understanding correspond to rules for synthesising the manifold of intuition. The concept 'chessboard', for instance, corresponds to a rule whereby this particular bit of white, that particular bit of black, etc. can be associated in one representation. When we synthesise, i.e. combine, some particular manifold of intuition according to the particular rule for a concept, we say that the manifold has been subsumed under the concept and cognised in accordance with it (A68-69/B93-94). A *pure* concept of the understanding is called a *category* (A80/B106); it is one of a set of 'meta-concepts' that all empirical concepts necessarily presuppose. Like the pure forms of intuition, the categories are a priori.

The application of a category to the manifold of intuition is governed by the *schema* of that category (A137-147/B176-B187); i.e. the rule by which the manifold is determined in accordance with the category. For example, the pure schema of magnitude is number (A142-143/B182). In apprehending the manifold corresponding to an object, determinate intuitions—'instants'—of time are produced with each successive act of synthesis. Through these is produced a *time series* from which one judges as to the extent of the object apprehended (A145/B184). Through an analysis of the use of the categories in accordance with their characteristic schema, we are entitled to make a number of synthetic a priori judgements regarding the objects of cognition in general. These are called synthetic a priori *principles* (A159-235/B198-294) by Kant.

Of particular importance for our discussion in the next and subsequent sections, wherein we will be considering the relevance of quantum theory for Kant's philosophy, is the principle of causality. Kant calls this a 'dynamical principle' since it is a principle governing the connection of appearances in time. It tells us, according to Kant, that the changes undergone by an object of cognition are ordered uniquely and objectively according to a necessary rule (A191/B236). This is in contrast to the series of subjective perceptions of the object through which we apprehend it (cf. A191-193/B236-238). Kant writes: "appearance, as contrasted with the [representations] of apprehension, can be [represented] as an object distinct from them only if it is subject to a rule that distinguishes it from any other apprehension and that makes necessary one kind of combination of the manifold" (B236/A191).[15] The particular rule of succession applicable to a given object is something that can only be discovered empirically. However, that cognition of an object *requires* the applicability of some such rule is what the principle of causality tells us. This is a priori, according to Kant. One notable aspect of such a rule, which we will return to later, is that Kant seems to require that it be deterministic in the sense of making possible the perfect *prediction* of an event from what has immediately preceded it. In Kant's words: "what precedes an event as such must contain the condition for a rule whereby this event always and necessarily follows" (B238–239/A193).

The concept of something that is an object *for us*—i.e., of something we have achieved objective cognition of—presupposes that the manifold corresponding to it has been determined in accordance with the principle of causality and the other synthetic a priori principles, for Kant (A159/B198). The flip side of this is that we are required to investigate nature in accordance with these principles, for it is only through them that the objective cognition of anything is possible. But it does not follow from this that the objects of our inquiries are already determined in accordance with these principles in advance of our investigations, or that we can know a priori that any particular endeavour to attain objective cognition will be successful (cf. A509/B537). A particularly striking example of the failure to cognise an object in this sense is provided by quantum theory, which we will begin to discuss in the next section.

## D Indeterminacy

Consider the following scenario.[16] A medium-sized object is launched towards a diaphragm into which an opening, or 'slit', has been cut, large enough for the object to pass through, but

[15] I have modified Pluhar's translation of *Vorstellung* as 'presentation' to the more standard 'representation'. As Allison (2001, p. 348) points out, Kant considered this term as equivalent to the Latin *repraesentatio* (A320/B376).

[16] The following example is adapted from Bacciagaluppi (2015, p. 381), as is my discussion of the analogous single-slit example involving quantum phenomena which follows. The quantum example is of course originally due to Bohr (1935). In his paper, Bohr actually discusses (successively) multiple variations of the single-slit experiment. The one discussed here and in Bacciagaluppi (2015) is the particular variation discussed on pp. 698–699: "In an arrangement suited for measurements of the momentum of the first diaphragm, it is further clear that even if we have measured this momentum before the passage of the particle through the slit, we are after this passage still left with a *free choice* whether we wish to know the momentum of the particle or its initial position relative to the rest of the apparatus".

small enough so that it is invariably deflected to some degree as it does so, before it finally impacts upon a screen some distance from the diaphragm. If the diaphragm is movable (e.g., attached to the rest of the apparatus by springs), it will recoil slightly as the object passes through it. We would like to describe the object's state immediately after this interaction.

Assume the (centre-of-mass) positions and momenta of both object and diaphragm just prior to the interaction are known. Further assume that we do not know the precise shape and size of the object. This will prevent us from calculating (in advance) what the respective positions and momenta of the object and diaphragm will be after the collision. But once they do collide, measuring the recoil of the diaphragm will determine its momentum, which will allow us to infer (via the conservation law for momentum) to the momentum of the object immediately after the collision. And since the common centre of mass of the combined system remains at rest, measuring the position of the slit will determine the object's position.

Within classical mechanics, the position and momentum of an object at a particular time together constitute a complete characterisation of its state at that time, which allows us to infer to its state at all other times (thus to where it will impact on the screen).[17] Note that a complete characterisation of the object's state can be had despite the fact that the diaphragm disturbs the object's motion. For in classical mechanics, either the interaction of an object with a measuring instrument can be made negligible for the purposes of a given analysis, or (as above) we can use physical theory to abstract away from or correct for this interaction. Indeed for precisely this reason, after we have measured the momentum of the *diaphragm*—which will in general disturb *it*—we can subsequently measure its position, using physical theory to correct for the interaction involved in our previous momentum measurement to determine the position of the object which has just passed through. In this way we are able to simultaneously ascribe *both* position and momentum parameters to the object (Bai & Stachel, 2017). In the words of the previous section, in every case classical theory allows us to transition from the subjective conditions (represented by the diaphragm in the above example) under which we perceive an object, to an objective description of that object wherein these subjective conditions no longer (explicitly) appear. This process of determination is characteristic of *all* objective cognition, as

[17]Note, however, the caveat mentioned in Hughes (1989, sec. 2.4).

we have seen, for Kant.

Things are more interesting in quantum mechanics. Consider, for instance, a quantum analogue of the above (schematic) example, similar in every respect except that the system of interest is now a photon. As above, we would like to determine its position and momentum immediately after it collides with the slit. In this case, however, quantum mechanics' well-known indeterminacy relation for position and momentum precludes us from ascribing determinate values to these quantities simultaneously; it tells us that as the indeterminacy in the position of an object approaches zero, the indeterminacy in its momentum approaches infinity, and vice versa. In terms of our example this means that if we choose to precisely determine the momentum of the diaphragm (so as to ascribe a determinate momentum to the photon), the diaphragm's displacement consequent upon that determination of it will, in contrast to the situation in classical theory, be "uncontrollable" (Bohr, 1935, p. 698)—or at any rate not controllable enough—and we will be unable to account for this displacement and correct for it as we did above for the purposes of a subsequent position determination of the photon. Similarly, if we instead choose to set up the experiment so as to make determinate the diaphragm's position (and thus the position of the photon), through this choice we will have precluded ourselves from precisely determining the photon's momentum.[18]

## E The challenge for Kant's philosophy

From the point of view of Section C, we can understand this as follows. Consider the result of some experiment, say the mark on a photographic plate, or the particular situation of a pointer measuring the momentum of the diaphragm in the above example. As such, the pointer and the mark are describable as classical objects in the sense that they can (effectively) be described in the ordinary way as having definite spatiotemporal coordinates, and as causally interacting in a

[18]Note that if we choose not to measure the diaphragm at all, then the joint state of the diaphragm and photon will be describable as an entangled quantum superposition. Hermann (2017, sec. 10) makes this point in the context of a different thought-experiment (the $\gamma$-ray microscope experiment). The role played by the diaphragm in our example is in that context played by a photon which is collided with an electron (the system of interest) in order to determine the latter's state. In place of a pointer connected to a diaphragm, we have in that context a photographic plate which can be set up in various ways (or not at all) in order to measure the state of the photon. See also fn. 26 below.

definite way with their surroundings. However our aim (which as we will see cannot be satisfied in quite the way one would like in the quantum context) is to go beyond the particular mark and the particular pointer reading and describe these as having arisen through the interaction of our experimental apparatus with some independently existing object. Our goal, in other words, is to 'get at' this object as it exists independently of the 'subjective conditions' associated with the particular experiments we subject it to. And the way we do this—or anyway the way we attempt to do it—is by compensating mathematically for the interaction between the apparatus and object in our description of the latter.

For a Kantian, it is of course never possible to *completely* eliminate in this way the subjective conditions pertaining to our observation of an object from our objective description of it, since any description of an object that is cognisable for us must be determinable with respect to space and time. This is perfectly acceptable, however, since our standard for objective cognition is anthropocentric rather than theocentric. And since space and time are the pure forms attaching to any possible experience, we can in principle achieve a kind of objectivity through the attainment of a description of the object which will be valid for all discursive cognisers such as ourselves.

Now in order to describe something objectively in this sense it must be determinable in space and time in accordance with the synthetic a priori principles. Earlier I mentioned causality as an example of a 'dynamical' principle of this kind. In addition to the dynamical principles there are also what Kant calls *mathematical* principles (B198–B294).[19] According to the latter, anything that appears to us must be apprehended as having, determinately, both an extensive (length, breadth, etc.) and an intensive magnitude (i.e. a degree). The dynamical principles, in contrast, are not principles for the apprehension but for the connection of appearances in time. They state, first, that all change presupposes something permanent; second, that all change must occur according to the law of cause and effect; third, that all substances that are perceived as simultaneous are in mutual interaction.[20]

[19]The mathematical principles are the *Axioms of Intuition* and *Anticipations of Perception*; the dynamical principles are the *Analogies of experience* and the *Postulates of empirical thought as such*.

[20]Here, I only consider the *Analogies*, as the *Postulates* are not directly relevant for our discussion.

An objective description in the above sense is one that is determined in accordance with *both* sets of principles. Together, they assert that the determination of any appearance as an object of possible experience must be such that at a determinate instant in time, the object has a determinate extent (constrained by the mathematical principles) and hence a determinate position in space, and that there is a law (subject to the dynamical principles) by which it dynamically interacts with its surroundings in and through time. In the context of our example of the slit, one can interpret this as signifying that any description of a quantum object that purports to pick out an object of possible experience for us must be such as to ascribe to that object both a determinate position and a determinate momentum parameter. But according to the indeterminacy relations, it is impossible in principle to describe the particle's momentum with any degree of precision without a corresponding loss of precision with regards to its spatial coordinates. The upshot of this is that we cannot complete our description of the object according to the Kantian criteria for objects of possible experience. And yet these are *necessary* criteria, for Kant, in the sense that, as we have seen, objective cognition is impossible for us without them. We may get around this in some sense by ascribing only indeterminate values of position, momentum, etc. to the 'object' of our investigations, but the resultant 'unsharply defined' description (cf. Bohr, 1928, p. 582) can as a result never be an object *for us*—i.e. it can never be an object which we can have possible experience of and thus never be real for us in that sense. We can consider it merely as a noumenon, or abstract object (see Cuffaro, 2010, p. 313).

For one seeking to defend the Kantian philosophical framework, which includes the Kantian synthetic a priori principles—and in particular the principle of causality—this is good news, at least in one sense. For despite the fact that the indeterminacy relations imply that in general it is impossible to predict with certainty the future behaviour of a quantum object from a complete characterisation of its present state, i.e., that the latter in general does not "contain the condition for a rule whereby [a subsequent] event always and necessarily follows" (B238–239/A193–194), Kant's synthetic a priori principles are not thereby invalidated. For quantum state descriptions are merely abstract objects from a Kantian point of view and thus are not objects of possible experience for us. It is only the concept of the latter which presupposes that it be determinable in accordance with the principle of causality and other synthetic a priori principles. And it is only for such objects of possible experience that the perfect prediction of its subsequent states from a determinate description of the object is implied. Quantum mechanics is in this sense consistent with Kant's philosophy.

There is, nevertheless, a potential problem, if not for Kant's framework as a whole then at least for the principle of causality. For as we saw, the flip side of its being presupposed by the very concept of objective cognition is that we are required to investigate nature in accordance with it, for it is only through the principle of causality and the other synthetic a priori principles that the objective cognition of anything is possible. But if objective cognition in the Kantian sense is typically excluded in quantum mechanics, and yet we somehow are able to do physics in spite of this (Bohr, 1937, pp. 291–293), then it seems that we can do away with causality as a principle even if, strictly speaking, it is not contradicted by quantum mechanics. The logical empiricist Moritz Schlick expressed this anti-Kantian objection in 1931 as follows:

> The principle of causality does not directly express a fact to us, say, about the regularity of the world, but it constitutes an imperative, a precept to seek regularity, to describe events by laws. Such a direction is not true or false but is good or bad, useful or useless. And what quantum physics teaches us is just this: that the principle is *bad*, useless, impracticable within the limits precisely laid down by the principle of indeterminacy (1962 [1931], p. 285).

This objection strikes deep, indeed to the very heart of Kant's philosophical project to provide a principled characterisation of objective cognition. For if the Kantian principles are inapplicable within a particular domain of physics, it can be but small comfort to know that they are (for that reason) still strictly speaking valid in general. Kant's epistemological grounding, this objection asserts, is irrelevant within this specific domain and of no use to us.

## F Hermann's response

Grete Hermann's reflections on quantum mechanics' significance for the 'critical philosophy'—the philosophical tradition begun by Kant and continued by Fries as well as by her own teacher, Nelson—are for the most part contained in a long essay she published in 1935 (Hermann, 2017).[21] She begins with the admission that the emergence of quantum theory has "shaken" (p. 239) this tradition and in particular the idea that a priori principles discoverable through critical philosophical analysis lie at the foundations of natural science. Ultimately she will argue that the challenge posed by quantum theory can be met, but it cannot be met simply by recapitulating the critical deduction that had previously been made of critical philosophy's content.[22] She writes:

> For, even if the physical development of the theory is not sufficient to put the foundations of the thus achieved knowledge of nature into the sharp light of awareness, still the scientific progress that has been obtained in these theories precisely through the willingness to abandon or revise old familiar concepts provides the guarantee that new and fruitful points of view have been introduced here into research. Only their philosophical interpretation and elaboration will produce clarity concerning both the philosophical arguments for the a-prioricity of natural-philosophical principles and the objections to them arising from the side of physics (Hermann, 2017, p. 240).

In looking to quantum theory to illuminate the critical philosophy in this way, Hermann's attitude differs starkly from Nelson's. Nelson, as we will see later, was far from denying the general idea that philosophical insight could be obtained through the analysis of scientific knowledge. However Nelson held that the physics of the early twentieth century (not just quantum theory) was still too immature and full of contradictions to provide the kind of insight required to illuminate the critical philosophy. He rather recommended that—for the time being—critical philosophers adopt "the self-denying stance of Conventionalism" (Nelson, 1971, p. 253) toward physics and consider it to be "of purely heuristic significance" (ibid.) for the purposes of critical philosophy. But while Hermann maintained that real insight into the a

[21]For more on Hermann's life and work, see the short biography immediately prior to this chapter. Some early commentaries on Hermann's thought (which we will not be discussing in this chapter) can be found in Feyerabend (2015, p. 33, fn. 2) and Jammer (1974, pp. 207–211).

[22]As we will see in more detail later, 'deduction' is being used here in a specific technical sense that is relevant to Friesian critical philosophy. Her essay was originally published in the Friesian journal *Abhandlugen der Fries'schen Schule*.

priori philosophical principles of natural science could be gained through a critical philosophical analysis of the challenge posed to them by quantum theory, she noted that this could not take the form suggested by physicists such as Born, of attempting to disprove quantum theory by empirical means so as to restore these principles to their former unexceptioned status (Born, 1929). This is because the question raised by quantum theory is whether such principles are a priori precisely in the sense of being amenable to a purely philosophical, albeit in some sense empirically informed, analysis (Hermann, 2017, pp. 240–241).

With respect to the critical philosophy's principle of causality in particular, Hermann distinguishes in it causality proper, i.e., causality understood as an explanatory principle,[23] from the criterion through which causal connections in nature can be known: the principle of perfect prediction. But she does not do so in order to minimise the latter's importance. She writes:

> One who wished to brush [off the challenge to causality from quantum mechanics] with the excuse that, while the *knowledge* of the causes determining the processes is limited, the *existence* of such causes is not put in doubt, removes the law of causality from the realm of the principles governing natural knowledge into that of mysticism. Where it is impossible in principle to decide what falls under a given concept in nature, the statement *that* anything falls under it also loses its meaning (Hermann, 2017, p. 242).

Hermann nevertheless cautions against the use of the criterion of prediction in the "positivistically distorted form" (ibid.) employed by certain philosophers of her day. Presumably she has in mind those, such as Schlick (1961 [1931], sec. 6), who simply identify causality with perfect prediction. Causality and perfect prediction are closely linked, so much so that previously the satisfaction of one had always been taken to entail the satisfaction of the other (Hermann, 2017, p. 261). But for Hermann quantum mechanics makes it clear that these concepts eventually come apart. The usefulness of the criterion of prediction is limited within quantum mechanics by the indeterminacy relations: because a quantum object cannot be si-

[23] Regarding the occurrence of a measurement outcome, she writes: "It would . . . be pointless to wish to seek the cause of its occurrence in new physical features hitherto overlooked . . . *The theory of measurement already contains a sufficient basis for explanation*" (p. 255, emphasis in original).

multaneously described as having a determinate position and momentum, its future motion is not determined by these state parameters as it would be for a classical object. But it does not, Hermann argues, follow from this that quantum objects do not exhibit causal connectedness.

One way to recover such causal connectedness is to posit further parameters, not described by quantum mechanics, that precisely determine an elementary object's motion, although Hermann insists that we should require such parameters to be testable, i.e., usable in principle to perfectly predict phenomena: "whoever pretends to know such features, ought to prove the supposed correctness of his assertion, by deducing from it correct predictions about the outcomes of measurements" (Hermann, 2017, p. 254). After showing that the so-called proofs demonstrating the impossibility of such testable parameters are unsound, Hermann writes: "there can be only one sufficient reason for abandoning as fundamentally useless the further search for the causes of an observed process: *that one already knows these causes*" (2017, p. 254).[24]

Hermann maintains, though, that quantum mechanics does not actually motivate the search for additional causes of the processes it describes, for quantum mechanics itself provides the resources with which to identify any such causes. She notes in particular that interpreting the physical situation of a measurement pointer as relating a fact about the current state of a quantum system of interest presupposes the validity of a (classical) theory of the interaction between the system and apparatus through which the particular situation of the pointer has arisen.

For instance, in the above example of the diaphragm and slit one appeals to the conservation law for momentum in order to interpret the reading of the pointer after one has measured the diaphragm as asserting a fact about the quantum object which has just passed through. But in making such an inference, Hermann argues, the reading of the pointer is thereby "explained as the necessary effect that the system to be measured has imposed on the instrument in the process of measurement" (Hermann, 2017, p. 255). In other words, we posit a fact about the quantum system of interest—that it has a particular momentum—as an explanation which, in

[24]For more on Hermann's refutation of von Neumann's proof in particular, see the contribution by Michiel Seevinck in the edited volume by Crull & Bacciagaluppi (2017), and also Dieks (2017, sec. 5).

light of the classical conservation theorems, accounts for the fact that the pointer reading has a particular value. Moreover such a posit is testable, for quantum theory predicts that, given this posit regarding the system of interest's momentum, a subsequent measurement of that parameter will (experimental error aside) yield the same value again with certainty.[25]

Because of this there is no need, Hermann claims, to seek for the physical features overlooked by quantum mechanics that would make possible a causal explanation of the measurement pointer's particular reading. For a (classical) causal explanation of this particular measurement result is already provided by quantum mechanics in the way just mentioned. Likewise, the classical theory of the interaction appealed to in this case provides a classical causal explanation for the particular value we ascribe to the system of interest itself subsequent to its interaction with the diaphragm. Nothing about this analysis changes essentially, moreover, if it is a microscopic object that plays the role for us of a measuring instrument (Hermann, 2017, p. 255), for example when we use a microscope to collide a photon with an electron in order to measure the state of the latter.[26]

## G Hermann's departure from Kant

In the example of the diaphragm and slit, the uncontrollable displacement of the diaphragm, consequent upon a measurement of its momentum, bars us from forming a posit regarding the system of interest's position immediately after it interacts with the slit. The upshot of this is a kind of 'one-sidedness' in our characterisation of a quantum system even though it exhibits causal connectedness. As Hermann puts it:

> the quantum mechanical description by which, on the basis of some observation, a physicist determines his system, does not characterise this system completely and absolutely, but (so to speak) reveals only one aspect of it—precisely the aspect that

[25]This is just the statement that (perfect) quantum measurements are repeatable.

[26] In sec. 10 of her exposition, Hermann explores the microscope example alluded to here at length. Besides being a concrete illustration of the more abstract discussion of sec. 9, sec. 10 additionally contains (among other things) one of the earliest discussions of quantum entanglement in either the physical or philosophical literature. For more on the particulars of Hermann's microscope example, see Filk (2017); Frappier (2017), and for a detailed comparison of it with the 'diaphragm and slit' example, see Bacciagaluppi (2017).

> presents itself to the researcher on the basis of the observation made here (2017, p. 256).

To clarify: for any experiment performed on a quantum system of this kind one can, in the way described above, appeal to a classical law by which the reading of our measurement instrument emerges as the necessary consequence of a posited feature of the quantum system measured. In that sense we classically explain the reading of the pointer via this posit and the relevant classical conservation law. However at least from a classical point of view such a description of the quantum system is not a fully objective one, in the sense that it leaves out information that we require for a classically complete characterisation of the system's state. As Léna Soler and others have correctly pointed out,[27] however, this presents one with a problem if one's goal is to defend a Kantian conception of causality.

Hermann thus seems to depart from Kant's conception in the end. For Kant a causal process is one in which the sequence of appearances of an object can be connected together in time. But the appearance of an object is an appearance that can be given an objective description. And an objective description is one in which *both* dynamical and spatiotemporal parameters can be ascribed to the object. Thus the causal explanations that Hermann claims can be reconstructed from our various observations of phenomena do not seem to be causal in a Kantian sense. One may doubt, therefore, whether Hermann's analysis of the situation in quantum mechanics, even if one deems it to be a success otherwise, can be understood as a vindication of Kant's critical philosophy. Indeed Soler calls this departure of Hermann's view from Kant's one of the weaknesses of her view.

To be fair to Hermann, though, she nowhere claims to completely adhere to the critical philosophy as it was expounded by Kant himself. Indeed, as we will see further below, she is critical of Kant in her essay. We will also see how the conception of causality that results from Hermann's analysis of quantum theory can be seen as a natural consequence of applying the critical method as it was elaborated by Fries and Nelson—as a natural extension of Friesian neo-Kantianism.

[27] See, especially, p. 65 of Soler (2017). Erik Banks (2017, p. 257) makes a similar point, and so does Bacciagaluppi (2017, p. 140), although in the latter case Bacciagaluppi's focus is on this difference between Hermann's view and Heisenberg's rather than Kant's.

## H Hermann's Friesian roots

Fries, like Kant, saw the goal of critical philosophy to be primarily one of explicating the a priori in cognition, understood (to use Allison's terminology) not in a theocentric but in an anthropocentric sense. Recall that for Kant, the nature of our cognition is discursive. By this he means that cognition is composed of both a sensible and a conceptual aspect—the faculties of sensibility and understanding, respectively. Kant also refers to the faculty of *reason*, and indeed dedicates the lion's share of the Critique (the Transcendental Dialectic) to this faculty. But by reason he means, according to Fries, merely dialectical reason, i.e., the ability to make inferences, which for Fries is not essentially different from understanding. For understanding is the faculty of judgement, and dialectical reason is merely a kind of judgement; together they comprise only a single faculty: the faculty of mediative or reflective reasoning. In addition to this faculty and the faculty of sensibility, Fries holds that there must be a third, autonomous, faculty involved in our cognition: pure reason properly speaking, which operates spontaneously and involuntarily and is, for Fries, the proper source of the a priori principles of our cognition (Leary, 1982, pp. 227–228).

These a priori principles, for Fries, are immediate, though they are not sensations. They are cognitive, and yet they are not a species of reflective cognition. They thus constitute a kind of *immediate cognition*, which when applied in particular instances manifests itself in what he calls a 'feeling-for-truth' (Nelson, 1971, p. 178–179). But because this cognition is not reflective, it is not amenable to a formal proof, such as the one given by Kant on the basis of the formal conditions for the possibility of experience in the first Critique. Rather, our immediate cognition may only be 'deduced'—i.e., discovered and described—through an analysis of the actual subjective contents of our cognition. In this sense the method of the critical philosophy is, according to Fries, *empirical*. Through a philosophical analysis of the data that is our subjective consciousness, we deduce the a priori forms of knowledge that are implicitly relied upon therein (Leary, 1982, pp. 228–229).[28] In this sense Fries's conception of critical philosophy can be thought of as, as Nelson puts it, a "combination of Kant's notion of the Critique of Reason with

[28]For a detailed account of Fries's method of deduction see Nelson 1971, pp. 164–196. We will be considering part of Nelson's exposition in more detail below.

Plato's idea that philosophical knowledge is fundamentally obscure" (Nelson, 1971, p. 160).

The reader will at this point be forgiven for thinking that Fries's critical philosophy is itself obscure, however. The idea of a 'feeling-for-truth' is unclear and seems to amount to a bald appeal to intuition characteristic of pre-Kantian philosophy or even of contemporary analytic metaphysics. In this sense it does not seem to have much in common with Kantian philosophy. More importantly the idea of an immediate cognition that, despite its immediacy, is only known obscurely seems incoherent (cf. Beiser, 2014, p. 78). And it seems problematic to hold that knowledge of the a priori can be grounded empirically. It is thus fortunate that one of Nelson's most important contributions to Friesian critical philosophy was to clarify these and other aspects of Fries's views. The distinction between the a priori forms of knowledge and the Friesian critical philosophy, Nelson explains, can be understood in terms of what we would today call the distinction between object-language and meta-language (Nelson, 1971, pp. 184–190): "The peculiarity of the Critique of Reason, as practised by Fries, lies in this: the knowledge contained in the system of metaphysics forms the object of the knowledge contained in the Critique of Reason" (Nelson, 1971, p. 184).

Nelson illustrates the distinction via a discussion of the axiomatic method in mathematics. In the system of mathematical knowledge that we call geometry, Nelson explains, one often needs to answer questions about the various statements in the system and their relations to one another. To do so one uses what we would now call a meta-language, in which we express propositions about the object-language (in this case the system of geometry) such as: '$\phi$ is unprovable within the system $\mathfrak{G}$' (where $\phi$ is a statement in the object language $\mathfrak{G}$), 'The minimal set of axioms for $\mathfrak{G}$ is $\{\phi, \psi, \chi\}$', and so on (Nelson, 1971, p. 185).

To illustrate how the knowledge encoded in the meta-language can be acquired empirically: suppose that, rather than being presented with the complete specification of some object-language, one is instead successively presented with a series of theorems in it. Then, as each new theorem is exhibited, one's understanding of the object language grows; we learn empirically, so to speak, about its characteristics, which we express in our meta-language.[29] This relationship between meta-language and object-language is entirely analogous, Nelson main-

[29]This illustration of how one may learn about an axiomatic system 'via observation' is not actually a part of Nelson's discussion, but it is fully in the spirit of that discussion.

tains (1971, p. 185), to the relationship between Fries's critical philosophy and the 'system of metaphysics'; i.e., the a priori forms of knowledge (which is *not* metaphysical cognition in the theocentric sense). But in place of mathematical statements, our object-language in this case contains statements such as '$B$: every change has a cause', while in the meta-language, analogously to the mathematical meta-language that contains statements such as '$\phi'$: $\phi$ is unprovable', the critical philosophy contains a statement asserting: '$B'$: $B$ recapitulates some item of immediate knowledge', where $B'$ is 'deduced' in the Friesian sense by way of the empirical knowledge we have acquired thus far.

Fries's tripartite conception of cognition amounts, as does the orthodox Kantian bipartite discursive conception, to a kind of ontological posit. However in both cases these are posits regarding the concept of cognition. They do not aim to make substantive assertions about the properties of objects in the mind-independent world (cf. Nelson 1971, p. 166, Beiser 2014, sec. 12). As for the a priori principles of cognition that one discovers through the Friesian critical process of deduction—the process that begins with the actual contents of cognition rather than with its mere form—these in the end turn out to be identical to those arrived at by Kant. It is worth noting in particular that the Friesian a priori principles, like Kant's, are spatiotemporal, for Friesians, like Kant, adopt the anthropocentric paradigm of cognition. They further take Kant to have correctly identified two of the elements involved in our cognition; he has only missed the crucial third, and thereby the key to understanding how the principles of cognition can be discovered by us. In this sense one may say that Fries's contribution to critical philosophy pertains not to the elaboration of the end result of critical investigation but rather to the method that is to be followed so as to arrive at that result. Moreover since this critical method is, for Fries, an empirical one, it seems that it cannot be ruled out that further investigation will uncover new a priori forms or even modifications in our understanding of the a priori forms we have already uncovered. *The critical method may in principle yield surprises.* And this, Grete Hermann maintains, is precisely what occurs when we re-examine the critical philosophy in the light of the discoveries of quantum mechanics.

### I Hermann’s reconceptualisation of transcendental idealism

Hermann begins her discussion of the deeper consequences for the critical philosophy of the discoveries of quantum mechanics in sec. 16 of her 1935 essay. She begins with the observation that the nature of the Kantian system of natural philosophical principles (i.e., Kant’s synthetic a priori principles) is intimately bound up with his doctrine of transcendental idealism, which she characterises as follows: “according to [transcendental idealism, the synthetic *a priori* principles] cannot provide adequate knowledge of reality ‘in itself’ but only a limited knowledge of nature that stops at the conceiving of ‘phenomena”’ (Hermann, 2017, p. 271). Kant’s own main argument for transcendental idealism (see Section B above) had proceeded from his identification of space and time as the pure forms of our intuition. But this argument is taken by Hermann (and by all Friesians) to have been unsuccessful.[30] She takes Kant’s *successful* argument for this thesis to have arisen from his discussion of the antinomies. As Nelson had explained, these arise “through our silently assuming that Space and Time could contain a Whole of all existing things” (1970, p. 170), and are resolved via the doctrine of transcendental idealism properly construed:

> We can only describe the phenomena perceived as far as our experience goes, though no limits are set to the progress of that experience. So there is indeed one Nature, in space and time, one network of phenomena, bound by necessary laws. But it is not a Universe, not an absolute Whole of all existing things. Knowledge of nature is knowledge only of phenomena (Nelson, 1970, p. 171).

In her own essay, Hermann presents a variation of this argument set in a modern physical context. She first considers the principle of cause and effect, and notes that it is not really possible, strictly speaking, to apply it even within classical physics, for the relation of cause and effect presupposes that we can distinguish between two contiguous states of an object, an

[30] Friesians (including Hermann in 1935) as a rule criticised Kant’s doctrine of formal idealism (see Hansen-Schaberg (2017, p. 6), and also Cuffaro (2018, sec. 10) for further discussion, as well as fn. 7 above). As Bacciagaluppi (Forthcoming) discusses, however, in her later career she backed away from this Friesian criticism of Kant, which she (rightly) came to view as a misinterpretation of his doctrine.

earlier and a later state. However the temporal evolution of a classical physical state is *continuous*, which in the formalism of classical physics is expressed by the fact that its laws take the form of differential equations. This means, however, that for every state of the system $\omega'$ that comes earlier than $\omega$, there is actually a state $\omega''$ in between $\omega'$ and $\omega$, and thus properly speaking "there are no temporally *contiguous* states and hence for no state of a system can one specify another that has directly brought it about or has been caused by it" (Hermann, 2017, p. 272, emphasis added).

To illustrate this, Hermann invites us to consider the 'acceleration field' which surrounds the Earth in the context of Newtonian gravitational theory. That is, Newtonian theory specifies a law that associates, with a given point in the space surrounding the Earth, a particular acceleration that will be felt by any body situated at that point (cf. Stein, 1967, p. 261). Thus one might say that this law causally links the presence of a body at that point (the 'cause') with the acceleration imparted to it there (the 'effect'). This acceleration, however, is defined via a differential equation, i.e. as the change in velocity experienced at the given point. 'Change in velocity', however, is not an independent physical process but rather a "*relation* that obtains between the instantaneous velocity and the subsequent course of nature" (Hermann, 2017, p. 272), i.e. the further points on the object's trajectory.

In fact it is impossible, Hermann argues, to determine the acceleration imparted to an object existing *at* a specific point in space *at* the specific instant that it is there, since by considering only this single (idealised) spatiotemporal point one can in no way determine the object's velocity. Rather one must consider a wider space-time region and then compute the change in velocity of the object within that region. In considering a wider region, however, we thereby go beyond the particular point at which the instantaneous acceleration imparted to the object has been attributed. Further, within this wider region the object will experience additional changes in acceleration as it moves about. So we see that in this case it is not really possible, strictly speaking, to isolate the effect (the acceleration) at a specific point for the purposes of attributing it to a cause (i.e., that it was present at that point).

We encounter similar difficulties, according to Hermann, when we try to characterise the state description for a classical system as an objective physical description in the strong sense of reflecting properties really possessed by that system. For in that case the physical quantities (mass density, velocity, etc.) attributed to the system via its state description will have to be be specified for every point comprising its extension as well as for every point along its trajectory. These specifications make use of differential equations, however, which as we just saw actually require extended spatiotemporal regions for their determination.

> One can thus say nothing as to the specification of the properties of physical objects or events that determine these as they are constituted in themselves; rather, the alleged properties of physical systems in truth only specify certain relations between the parts of the system, without these parts being themselves unambiguously specifiable (Hermann, 2017, p. 273).

We see, therefore, that even in classical physics the conception of things that exist in space and time and that stand in causal relationships to one another is in fact of only limited applicability; we can in principle successfully describe phenomena in accordance with this conception, but in the limit this conception breaks down. That is,

> As long as one is dealing only with finite, spatially and temporally extended physical systems whose own inner structure need not be considered beyond a certain limit, one escapes the difficulties of the limiting process that make impossible the full application of the concepts of cause and of substance (Hermann, 2017, p. 273).

In this sense one can employ the fundamental concepts of 'substance', 'causality', etc. within physical theory only if one understands one's use of these concepts in this context in terms of an 'analogy' (Hermann, 2017, p. 273). For a complete specification of the object in spatiotemporal terms is on the basis of the above considerations impossible. But as analogies, i.e., in these concepts' application to finitely specified objects—objects for which everything beyond a certain specified bound has been 'cut' away from our description of them—these fundamental concepts can be used fruitfully, and indeed are indispensable. For physical investigation always begins with the observation of a finite extended system which we conceive of as an incompletely specified thing existing in space and time in interaction with its surroundings. And it is the very fact that this specification is incomplete that presents us with the occasion for better determining the object of our investigations beyond our arbitrary initial 'cut' (Hermann, 2017, p. 274).[31] And indeed we do so. And yet although we can move this cut further and further forward we can never push it away entirely, for it alone guarantees that we can fruitfully describe the object of our investigation as a thing, albeit only by way of analogy; i.e., not as a thing considered in itself apart from the conditions under which it is represented by us, but always only in relation to the spatiotemporal limitation imposed upon our representation of it by the forms of our sensibility.

One might worry that this doctrine seems to depart substantially from Kant's. For Kant, recall, it is constitutive of what it means to be an object that it is fully determined in space and time in accordance with the mathematical and dynamical synthetic a priori principles. Hermann seems to be arguing, however, that in fact a full determination of this kind is impossible. But from that it would seem to follow that Kantian objects are strictly speaking impossible as well. Now while it is true that, as we have seen, for Kant the concept of an object of cognition presupposes the ascription to it of determinate spatiotemporal parameters, and while it is also true for Kant that our representations of space and time are of continuous infinitely divisible quantities (B211, B255), it is *not* a part of Kant's doctrine that an object must be determinable beyond limit. This would only follow if we took our spatiotemporal characterisation of the object to pertain to it as it is in itself. In that case a truly objective characterisation of the object would indeed require us to describe it exhaustively via a completed infinity of spatiotemporal divisions. Kant's synthetic a priori principles, however, are only applicable to appearances, and completed infinities of this kind are not to be found in experience. To assume so can result only in antinomies (A523–527/B551–555; see also Bell (2017, sec. 4)). This is the argument for transcendental idealism that Hermann and other neo-Friesians took to be successful, and which was re-expressed by Hermann in the language of classical physics as above.

Hermann points out that one might respond to this line of argument as follows (Hermann, 2017, p. 275): one might concede that the conception of things existing objectively in space and time must dissolve, if pushed to its inevitable logical conclusion, into a network of spatiotemporal relations that *cannot* be characterised in terms of such things, and therefore that

[31] Cf. Harper's (2011) account of 'theory-mediated measurements'.

the strong sense of the conception is impossible. Nevertheless one might maintain that it is possible that further investigation into the natural world will reveal that this spatiotemporal relational network is objectively determinable via a consideration of things describable in space and time in the weaker sense.[32] Far from disagreeing, Hermann maintains that this question can only be decided by empirical research; and that in fact it has actually been answered in the negative by quantum theory. For quantum theory shows us that our knowledge of this relational network is fundamentally *split*. Hermann (2017, p. 275) writes:

> [Quantum mechanics] does away with the notion that these relational networks should be determined at any rate through objective circumstances of things in space and time, and shows them in turn to depend on the manner in which the observer obtains knowledge of the system (Hermann, 2017, p. 275).

Our ascription of a causal history to a quantum object always depends, as we have seen, on the experimental context through which it has been determined. In the experiment, described earlier, for determining the momentum of a given quantum system, for instance, we explain the fact that the pointer took on a particular reading by positing that the system had a particular momentum at the time of its interaction with the diaphragm. That is, this posit, in conjunction with the classical conservation theorems, accounts for the fact that the pointer reading took on a particular value in the context of this measurement. In performing such a measurement, however, we cut ourselves off from ascribing a determinate value of position to the quantum system at the time of its interaction with the diaphragm, for the determinateness of our attribution of position to the quantum system varies inversely with the determinateness of our attribution of momentum. Further, not only does what we learn about the quantum system depend on our observational context, it is also the case that (see Section E above) these observational contexts are in general incompatible. Yet as Hermann points out, "the limit up to which one or the other model finds application is *not itself an objective property of the object*" (Hermann, 2017, p. 270, emphasis added).

[32]It is possible that the view represented by the hypothetical objector here may actually be that of Nelson. In the context of practical philosophy, for instance, Nelson maintained that the moral law, although only discoverable empirically and 'subjectively', was nevertheless objective. This was a conclusion with which Hermann disagreed (Leal, 2017).

Within quantum mechanics there is thus no viewpoint from which one can give the quantum object a complete characterisation in the classical sense of determining both its position and its momentum; it is therefore not an object in that sense. Nevertheless it is possible to speak objectively about the quantum object *from within* a particular observational context. We can say, that is, that (experimental error notwithstanding) *the fact that the quantum object had a particular momentum* was the cause of the pointer's having indicated a particular value, because there is no information which we lack in order to complete our characterisation of this object from the perspective of that experimental context. In other words we have a complete characterisation of 'the object'—i.e., the photon's momentum—that our experiment was set up to ascertain. And because a perfect measurement, when repeated, will according to quantum mechanics yield the same result, our posit concerning this object is testable.

## J Hermann's Kantianism

To come back now to the question we first considered in Section F: is Hermann's view actually Kantian? In fact there is a sense in which it is Kantian and a sense in which it is not. Let us begin with the latter: Kant has developed his system a priori, beginning from a posit regarding the nature of our cognition as involving the contributions of both the faculties of sensibility and understanding, and has provided an account of the a priori forms of these faculties (space and time and the categories). On their basis he has established a number of synthetic a priori principles which are presupposed by the concept of objective cognition as such. In the context of physical theory these principles imply that the objective determination of something must be such as to ascribe to it determinate values for both its position and its momentum. *Pace* Hermann, the one-sided 'objects' described in the previous section do not satisfy this criterion, however, and in this sense they are not objects of theoretical cognition to which the principle of causality could be applicable.

In a different sense, however, Hermann's viewpoint is indeed quite Kantian. For after all, Kantian philosophy teaches us that there are multiple ways of considering phenomena. We may consider ourselves as either objects of theoretical or practical inquiry, for instance, and each of these domains of inquiry comes with its own criteria for cognition. Moreover since our cognition of objects is limited to appearances it does not follow that any one domain can be thought of as either ontologically superior or reducible to the other (cf. Allison (2004, p. 48), Nelson (1971, p. 247ff)). Further, within each domain distinct notions of justification are operative, along with distinct notions of causality—most generally thought of as change in accordance with law (Kant, 1981, 4:446)—and the other fundamental concepts of the Kantian system. “The will is a kind of causality”, Kant writes, “belonging to living beings insofar as they are rational” (Kant, 1981, 4:446).

What quantum mechanics teaches us, according to Hermann, is that even *within* a single domain of inquiry, such as physical science, one can be confronted with a splitting of perspectives in the above sense:

> the splitting of truth goes deeper than philosophy and natural science had previously assumed. It penetrates into the physical knowledge of nature itself; instead of merely delimiting its scope against other possibilities for grasping reality, it separates various equally legitimate representations within the physical description that cannot be unified into a single picture of nature (Hermann, 2017, p. 277).

Beginning from such motivations it might seem natural for a Kantian to want to relativise the concept of causality to different perspectives in this way within the domain of physical science. And yet for an orthodox Kantian the way seems barred: the pure forms of our sensibility and understanding are what they are, and on their basis we elaborate a formal account of the constraints they impose on our conception of nature. The framework described by these constraints is static, both in terms of its form and in terms of the objects it relates: objects always possess *both* mathematical and dynamical attributes and are determinable in accordance with the categories. There is no room for a one-sided object in this scheme. Hermann, however, is not an orthodox Kantian but a Friesian neo-Kantian. And as we saw in Section H, for Friesians the method of critical philosophy is not a priori but *empirical*: It does not begin from the pure form of sensibility but from the *actual* contents of sensible cognition. Friesians are allowed to learn and adapt the critical philosophy in the face of the challenges posed to it by quantum mechanics in a way that orthodox Kantians seemingly cannot. The way is open, therefore, for Hermann’s relativisation of the concept of causality (and along with it, objectivity) within physical science. And she need not deny the central tenets of critical philosophy as it was understood by the Friesian school—namely its core conception of the critical *method*—in order to do so. Indeed Hermann's revision of the concept of an object to which the principle of causality could apply is thoroughly Friesian in spirit: it emphasises, after all, that the principles of cognition *can only* be uncovered through a process of reflection on the contents of our *actual experience*.

## K Conclusion

Let us now summarise our discussion and conclude. I introduced Kant's doctrine of transcendental idealism as an epistemic doctrine in Section B, noting that it rests on an ontological posit of a sort: the discursivity thesis, motivated for Kant by his struggle to come to grips with the possibility of metaphysical cognition in the period leading up to and including the publication of his inaugural dissertation. In Section C we discussed how Kant's search for the first principles of metaphysical cognition was transformed, during the critical period, into the search for the first principles of synthetic a priori cognition. In Sections D–E we discussed the challenges faced by Kant's view which arise from the emergence of quantum theory, and in Sections F–J we discussed Grete Hermann's assessment of these challenges to Kantian critical philosophy. We saw that Hermann is led to conclude that physical knowledge is fundamentally 'split', and how Hermann's solution to the problem of objectivity in quantum mechanics is rooted in the neo-Friesian tradition of which she partook.

Hermann's is not the only way to come to grips with quantum mechanics from a broadly Kantian point of view. Ernst Cassirer (1956 [1936]), whose intellectual origins are in Marburg neo-Kantianism, also rejects Kant's discursivity thesis but in a different way: the problem is not, as it is with Fries, that the discursivity thesis is incomplete; the problem for Cassirer is with Kant's understanding of intuition (p. 166). Bohr never called himself a Kantian, but his philosophy of quantum mechanics nevertheless begins from essentially Kant's understanding of our fundamental conceptual framework as discursive (Cuffaro, 2010). From this point of view, quantum mechanics can be seen to emphasise the basic point of Kant's transcendental idealism: that the epistemic primacy of the conditions under which we can assert that we have cognised some object does not imply the ontological primacy of those conditions (Bohr 1928, p. 580, Bohr 1949, p. 211). Yet in the face of this Bohr enjoins us to conceive of our basic conceptual framework pragmatically (cf. Faye, 2017; Folse, 2017), to make imaginative use of the resources provided by it, and to thereby extend its reach (Bohr, 1958, p. 88).

A more thorough comparison of the views of all of these neo-Kantian thinkers will have to wait for another occasion. But in the meantime we can say this in regards to Hermann's view. For Grete Hermann, as we have seen, the lesson of quantum mechanics is that the fundamental physical concept of objectivity includes a reference to a particular perspective, to the contents of an *actual* spatiotemporal experience, tied as all such experiences are to a particular measurement context, within which we may continue to meaningfully employ principles, such as causality, in our representations of the phenomena confined to that context. Quantum mechanics teaches us this, on Hermann's view. And it is a merit of her viewpoint that it makes straightforward sense of the fact that quantum mechanics can teach us *anything at all* about the a priori forms of our cognition. Moreover it hints, perhaps, at a more general account of the way that the a priori forms of our cognition—what in another context have been called 'framework assumptions'—may change their scope and meaning over time.

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