Information Causality, the Tsirelson Bound, and the 'Being-Thus' of Things

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What distinguishes QM from CM?

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Are quantum correlations the strongest possible no-signalling correlations?

- No ('PR' systems: Popescu & Rohrlich 1994)
 - ∃ conceivable stronger-than-quantum correlations that are non-signalling
- Why don't we see these correlations in nature?

'Information causality' principle (Pawłowski et al., 2009)

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- 'implausible accessibility of remote data' (Pawłowski et al., 2009, ibid.), 'things like this should not happen' (Pawłowski & Scarani, 2016, p. 429).

How to (begin to) motivate information causality?

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- 'implausible accessibility of remote data' (Pawłowski et al., 2009, ibid.), 'things like this should not happen' (Pawłowski & Scarani, 2016, p. 429).
- Expresses a generalised methodological sense of Einstein separability, suitable for a theory of communication

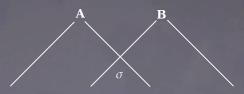
Outline

- PR-correlations
- Alice and Bob play a guessing game
- Information causality
- Motivating information causality
- Objections and larger questions

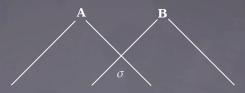
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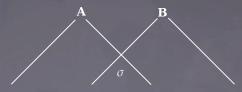
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- $\sigma\!\!:$ correlated state of two two-level subsystems

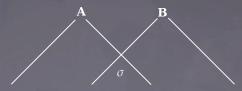


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- a, a', b, b': measurement settings



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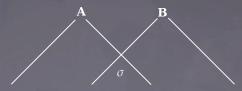
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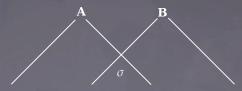
p(1,1|a,b) = 1/2 p(same|a,b') = 1 p(same|a',b) = 1 p(different|a',b') = 1.



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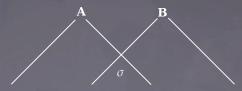
p(-1,-1|a,b) = 1/2 p(same|a,b') = 1 p(same|a',b) = 1 p(different|a',b') = 1.



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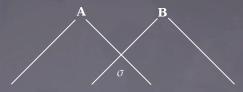
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'Non-signalling': p(A|a, b) = p(A|a, b')

$$\mathsf{P}(\mathsf{successful sim}) = \frac{1}{2} \left(1 + \frac{\langle a, b \rangle + \langle a, b' \rangle + \langle a', b \rangle - \langle a', b' \rangle}{4} \right)$$

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Why the Tsirelson Bound?

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Why the Tsirelson Bound?

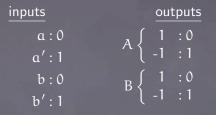
· Pawłowski et al. (2009): 'Information Causality'

Outline

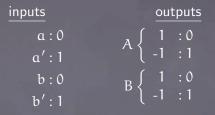
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<u>inputs</u> a:0 a':1 b:0 b':1

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a : 0	λ 5	1 :0 1 :1
a':1	A (-	1:1
b:0	B	1 :0 1 :1
b':1	D	1:1

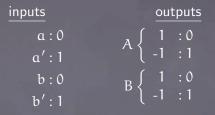


PR-system correlations: $a \times b = A \oplus B$

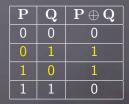


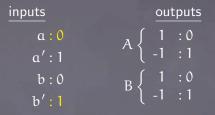
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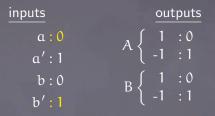




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E.g., '01' experiment:

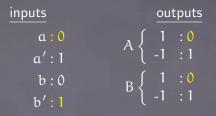
Ρ	\mathbf{Q}	$\mathbf{P} \oplus \mathbf{Q}$
0	0	0
0	1	1
1	0	1
1	1	0



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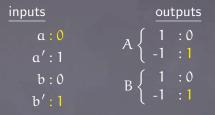
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 - $\cdot\,$ e.g., $\vec{b}=11\Rightarrow$ Bob guesses Alice's 3rd bit

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Alice:

- receives $\vec{a} = a_3 a_2 a_1 a_0$
- $\label{eq:alpha} \bullet \mbox{ measures } a_0 \oplus a_1 \mbox{ on box} \\ I, \Rightarrow \mbox{ gets } A_I$
- measures $a_2 \oplus a_3$ on box II, \Rightarrow gets A_{II}
- measures

 $\begin{array}{l} (\mathfrak{a}_{o}\oplus A_{I})\oplus (\mathfrak{a}_{2}\oplus A_{II}) \\ \text{on box III,} \Rightarrow \text{gets } A_{III} \end{array}$

• sends $c = a_0 \oplus A_I \oplus A_{III}$ to Bob Bob:

- receives $\vec{b} = b_1 b_0$
- measures b_0 on both box I and box II, \Rightarrow gets B_I and B_{II}
- if $\vec{b} = 00$ or $\vec{b} = 01$,
 - $\cdot \ \, \text{guesses} \ \, c \oplus B_{III} \oplus B_{I}$

• if
$$\vec{b} = 10$$
 or $\vec{b} = 11$,

 $\begin{array}{c} \cdot \;\; \mathsf{guesses} \\ c \oplus \mathsf{B}_{\mathrm{III}} \oplus \mathsf{B}_{\mathrm{II}} \end{array}$

Winning strategy for any N

Share N - 1 PR-systems:

 Bob can guess the value of <u>any</u> single bit of Alice's data set with certainty Winning strategy for any N

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What aboout general no-signalling systems?

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In general, probability of a successful answer by Bob in the game (for $N = 2^n$):

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• Forces E to be bounded by $\frac{1}{\sqrt{2}}$ (higher values violate I.C.)

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Subtleties:

- not all quantum correlations exceed T.B.
- Evidence that correlations within T.B., but stronger than quantum, satisfy I.C. (Navascués et al., 2015).

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- How simple is 'too simple'?

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Can a better motivation be given?

Outline

- PR-correlations
- Alice and Bob play a guessing game
- Information causality
- Motivating information causality
- Objections and larger questions

How to motivate information causality?

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 - \cdot 'mutually independent existence'

Einstein on Mutually Independent Existence (MIE)

"Without ... an assumption of the mutually independent existence (the 'being-thus') of spatially distant things, an assumption which originates in everyday thought, physical thought in the sense familiar to us would not be possible." (Einstein, translated in Howard 1985, p. 187)

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"... every statement regarding S_2 which we are able to make on the basis of a complete measurement on S_1 must also hold for the system S_2 if, after all, no measurement whatsoever ensued on S_1 " (ibid.)

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- 'surface-level' constraint (i.e. on the measurable properties of a system)





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 Mutually Independent Existence (MIE) best thought of as a methodological principle ⇒ a constraint on physical practice



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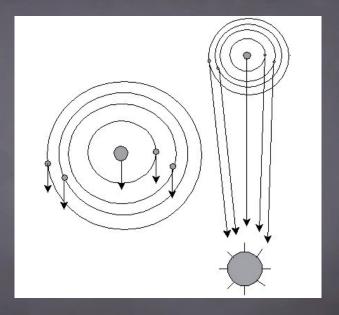


Figure: 1. Source: DiSalle (2016)



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- 'Judo-like manoeuvre': MIE is (in this sense) satisfied in quantum mechanics (no-signalling)

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For N = 4, use three PR-boxes: I, II, III

Alice:

- receives $\vec{a} = a_3 a_2 a_1 a_0$
- $\label{eq:alpha} \bullet \mbox{ measures } a_0 \oplus a_1 \mbox{ on box} \\ \mathsf{I}, \Rightarrow \mbox{ gets } A_I$
- measures $a_2 \oplus a_3$ on box II, \Rightarrow gets A_{II}
- measures

 $\begin{array}{l} (\mathfrak{a}_{o}\oplus A_{I})\oplus (\mathfrak{a}_{2}\oplus A_{II}) \\ \text{on box III,} \Rightarrow \text{gets } A_{III} \end{array}$

• sends $c = a_0 \oplus A_I \oplus A_{III}$ to Bob Bob:

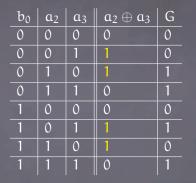
- receives $\vec{b} = b_1 b_0$
- measures b_0 on both box I and box II, \Rightarrow gets B_I and B_{II}
- if $\vec{b} = 00$ or $\vec{b} = 01$,
 - $\cdot \ \, \text{guesses} \ \, c \oplus B_{III} \oplus B_{I}$
- if $\vec{b} = 10$ or $\vec{b} = 11$,

 $\begin{array}{l} \cdot \mbox{ guesses} \\ c \oplus B_{III} \oplus B_{II} = \\ a_2 \oplus (b_0 \times (a_2 \oplus a_3)). \end{array}$

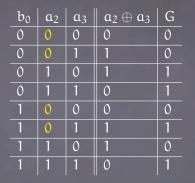
bo	a ₂	a3	$\mathfrak{a}_2 \oplus \mathfrak{a}_3$	G
0	0	0	0	0
0	0	1	1	0
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0	1	1	0	1
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 $p(G = 0 | \mathbf{a}_2 \oplus \mathbf{a}_3 = \mathbf{0}) = 1/2$

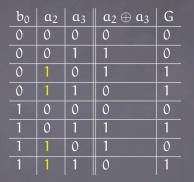


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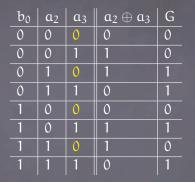
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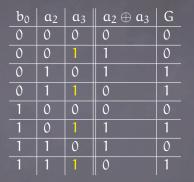
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- Does information causality express a form of mutually independent existence?

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 - · Non-extreme case: Bob has <u>locally accessible</u> information about $\vec{\alpha}$ over and above what's been transmitted to him
 - Extreme case: Alice's bits 'may as well be Bob's' for the purposes of the game:
 - Strategy is just as effective whether Alice's bits are localised with her or with Bob.

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- in this sense similar to Thermodynamics (Myrvold, 2011; Wallace, 2014; Ladyman, 2018)

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- How to make the practice of physics possible

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- One bit of communication needed to win \Rightarrow trivial (communicationally)

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- $N - 1 = 2^n - 1$ PR-systems required \Rightarrow exponential / 'hard' (computationally)

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Analogy: Corollary VI

Desirable:

- Show that degree of violation of IC (hence of TB) \propto 'degree of triviality'

Why should nature care whether we can do physics?

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No reason why it should care. But $\underline{in fact}$ we do have a science of physics.

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Mutually independent existence (and thus information causality) can be thought of as aiming to answer the question: <u>how are such</u> facts possible?

- not an 'appeal to intuition'

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- aim to identify the necessary suppositions implicit in <u>any</u> such theories and in our practice of them
 - \cdot what is required for empirical testing to be possible?
 - what distinctions must we make in order to quantify communicational resources?

But maybe not definitive answers! - Howard (1989)

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- Alternate lesson from analysis of IC/TB :
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 - Distinction between distributed and localised computational tasks becomes 'blurry' / 'confused' for the purposes of a complexity-theoretic analysis if IC is violated.
 - IC as a physically motivated constraint on mutual independence of two mathematical theories

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