# Information Causality, the Tsirelson Bound, and the 'Being-Thus' of Things 

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## What distinguishes QM from CM?

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- No ('PR' systems: Popescu \& Rohrlich 1994)
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Are quantum correlations the strongest possible no-signalling correlations?

- No ('PR' systems: Popescu \& Rohrlich 1994)
- $\exists$ conceivable stronger-than-quantum correlations that are non-signalling
- Why don't we see these correlations in nature?
'Information causality' principle (Pawłowski et al., 2009)
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- Can be used to derive the 'Tsirelson bound' (maximal value of quantum correlations)
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How to motivate information causality?

- World would be 'too simple' (Pawłowski et al., 2009, p. 1101), 'too good to be true' (Bub 2012, p. 180, Bub 2016, p. 187);
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How to motivate information causality?

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- 'implausible accessibility of remote data' (Pawłowski et al., 2009, ibid.), 'things like this should not happen' (Pawłowski \& Scarani, 2016, p. 429).
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How to (begin to) motivate information causality?

- World would be 'too simple' (Pawłowski et al., 2009, p. 1101), 'too good to be true' (Bub 2012, p. 180, Bub 2016, p. 187);
- 'implausible accessibility of remote data' (Pawłowski et al., 2009, ibid.), 'things like this should not happen' (Pawłowski \& Scarani, 2016, p. 429).
- Expresses a generalised methodological sense of Einstein separability, suitable for a theory of communication


## Outline

- PR-correlations
- Alice and Bob play a guessing game
- Information causality
- Motivating information causality
- Objections and larger questions


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p(\text { same } \mid a, b) & =1 \\
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'Non-signalling': $p(A \mid a, b)=p\left(A \mid a, b^{\prime}\right)$

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P(\text { successful } \operatorname{sim})=\frac{1}{2}\left(1+\frac{\langle a, b\rangle+\left\langle a, b^{\prime}\right\rangle+\left\langle a^{\prime}, b\right\rangle-\left\langle a^{\prime}, b^{\prime}\right\rangle}{4}\right)
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Why the Tsirelson Bound?

- Pawłowski et al. (2009): 'Information Causality'


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## Abstract notation:

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| :---: | :---: | :---: |
| F | F | F |
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- e.g., $\vec{b}=11 \Rightarrow$ Bob guesses Alice's 3rd bit


## Winning strategy for $\mathrm{N}=2$

## Alice and Bob share one PR-system

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Alice:

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& =a_{0} \oplus\left(a_{0} \oplus a_{1}\right) \times b_{0}
\end{aligned}
$$

- Suppose $b_{0}=0$ : Then $a_{0} \oplus\left(a_{0} \oplus a_{1}\right) \times b_{0}=a_{0}$
- Suppose $b_{0}=1$ : Then $a_{0} \oplus\left(a_{0} \oplus a_{1}\right) \times b_{0}=$ $\left(a_{0} \oplus a_{0}\right) \oplus a_{1}=a_{1}$


## Winning strategy for $\mathrm{N}=2$

Alice and Bob share one PR-system

Alice:

- receives $\vec{a}=a_{1} a_{0}$
- measures $a_{0} \oplus a_{1}$ on her subsystem
- gets result A
- sends $c=a_{0} \oplus A$ to Bob

Bob:

- receives $\vec{b}=b_{0}$
- measures $b_{0}$ on his subsystem
- gets result B
- guesses:

$$
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Alice:

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$\mathrm{I}, \Rightarrow$ gets $A_{\mathrm{I}}$
- measures $a_{2} \oplus a_{3}$ on box $\mathrm{II}, \Rightarrow$ gets $A_{\mathrm{II}}$
- measures
$\left(a_{0} \oplus A_{I}\right) \oplus\left(a_{2} \oplus A_{I I}\right)$ on box III, $\Rightarrow$ gets $A_{\text {III }}$
- sends $c=a_{0} \oplus A_{I} \oplus A_{\text {III }}$ to Bob

Bob:

- receives $\vec{b}=b_{1} b_{0}$
- measures $b_{0}$ on both box I and box II, $\Rightarrow$ gets $\mathrm{B}_{\mathrm{I}}$ and $\mathrm{B}_{\mathrm{II}}$
- if $\vec{b}=00$ or $\vec{b}=01$,
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- if $\vec{b}=10$ or $\vec{b}=11$,
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Winning strategy for any N
Share N-1 PR-systems:

- Bob can guess the value of any single bit of Alice's data set with certainty

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What aboout general no-signalling systems?

Recall: probability of successfully simulating a single PR-system:

$$
P(\text { successful } \operatorname{sim})=\frac{1}{2}\left(1+\frac{\mathrm{CHSH}}{4}\right)
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In general, probability of a successful answer by Bob in the game (for $N=2^{n}$ ):

$$
\frac{1}{2}\left(1+E^{n}\right)
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## Outline

- PR-correlations
- Alice and Bob play a guessing game
- Information causality
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- $\mathrm{E}={ }_{\mathrm{df}} \mathrm{CHSH} / 4$

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Subtleties:

- not all quantum correlations exceed T.B.
- Evidence that correlations within T.B., but stronger than quantum, satisfy I.C. (Navascués et al., 2015).

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Motivation?

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- "implausible accessibility of remote data" (ibid.)
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Can a better motivation be given?

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- PR-correlations
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- 'mutually independent existence'


## Einstein on Mutually Independent Existence (MIE)

"Without ... an assumption of the mutually independent existence (the 'being-thus') of spatially distant things, an assumption which originates in everyday thought, physical thought in the sense familiar to us would not be possible." (Einstein, translated in Howard 1985, p. 187)

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"... every statement regarding $S_{2}$ which we are able to make on the basis of a complete measurement on $S_{1}$ must also hold for the system $S_{2}$ if, after all, no measurement whatsoever ensued on $S_{1}$ " (ibid.)

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- 'surface-level' constraint (i.e. on the measurable properties of a system)


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- Mutually Independent Existence (MIE) best thought of as a methodological principle $\Rightarrow$ a constraint on physical practice


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Figure: 1. Source: DiSalle (2016)

Demopoulos (In Preparation):

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- 'Judo-like manoeuvre': MIE is (in this sense) satisfied in quantum mechanics (no-signalling)

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## For $\mathrm{N}=4$, use three PR-boxes: I, II, III

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$$
\begin{aligned}
& c \oplus \mathrm{~B}_{\mathrm{III}} \oplus \mathrm{~B}_{\mathrm{II}}= \\
& \mathrm{a}_{2} \oplus\left(\mathrm{~b}_{0} \times\left(\mathrm{a}_{2} \oplus \mathrm{a}_{3}\right)\right) .
\end{aligned}
$$

| $b_{0}$ | $a_{2}$ | $a_{3}$ | $a_{2} \oplus a_{3}$ | $G$ |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 |


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| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 |
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| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 |

$$
p\left(G=0 \mid a_{2} \oplus a_{3}=0\right)=1 / 2
$$

| $b_{0}$ | $a_{2}$ | $a_{3}$ | $a_{2} \oplus a_{3}$ | $G$ |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 |

$$
p\left(\mathrm{G}=0 \mid \mathrm{a}_{2} \oplus \mathrm{a}_{3}=0\right)=1 / 2=\mathrm{p}\left(\mathrm{G}=0 \mid a_{2} \oplus a_{3}=1\right)
$$

| $b_{0}$ | $a_{2}$ | $a_{3}$ | $a_{2} \oplus a_{3}$ | $G$ |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
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| 1 | 1 | 1 | 0 | 1 |

$$
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$$

$$
p\left(G=0 \mid a_{2}=0\right)=3 / 4
$$

| $b_{0}$ | $a_{2}$ | $a_{3}$ | $a_{2} \oplus a_{3}$ | $G$ |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 |

$$
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$$

$$
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$$

| $b_{0}$ | $a_{2}$ | $a_{3}$ | $a_{2} \oplus a_{3}$ | $G$ |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 |
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$$

$$
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$$

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| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
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How does information causality generalise no-signalling?

- In two senses:

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- "... there can be no information about the bits in Alice's data set in the previously established correlations themselves" (Bub, 2012, p. 180)

Mission accomplished?

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- Information causality generalises no-signalling to the case of communicating agents.

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- Does information causality express a form of mutually independent existence?

Bub's gloss on IC: "there can be no information [provided to Bob] about the bits in Alice's data set in the previously established correlations themselves" (Bub, 2012, p. 180)

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- Non-local joining at the operational level
- Non-extreme case: Bob has locally accessible information about $\vec{a}$ over and above what's been transmitted to him
- Extreme case: Alice's bits 'may as well be Bob's' for the purposes of the game:
- Strategy is just as effective whether Alice's bits are localised with her or with Bob.

Ok, but is MIE really necessary in this context?

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- in this sense similar to Thermodynamics (Myrvold, 2011; Wallace, 2014; Ladyman, 2018)

Our general approach so far

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How to motivate mutually independent existence?

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How to motivate mutually independent existence?

- How to make the practice of physics possible


## Our general approach so far

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How to motivate no-signalling?

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How to motivate no-signalling?

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How to (begin to) motivate information causality:

## Our general approach so far

How to motivate mutually independent existence?

- How to make the practice of physics possible

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How to (begin to) motivate information causality:

- How to make the practice of communicational complexity / information theory, as a 'practical' science / resource theory, possible

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- Unclear that a science of communicational complexity / information is really possible under these circumstances

Bub's gloss on IC: "there can be no information [provided to Bob] about the bits in Alice's data set in the previously established correlations themselves" (Bub, 2012, p. 180)
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MIE satisfied in QM/QIT (at the operational level):

- No-signalling
- Information causality


## Outline

- PR-correlations
- Alice and Bob play a guessing game
- Information causality
- Motivating information causality
- Objections and larger questions

Computational complexity vs. communicational complexity

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- One bit of communication needed to win $\Rightarrow$ trivial (communicationally)

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- One bit of communication needed to win $\Rightarrow$ trivial (communicationally)
- $N-1=2^{n}-1$ PR-systems required $\Rightarrow$ exponential / 'hard' (computationally)
'A little' ambiguity ok?
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Recall:

- Trivialisation results
- For PR-systems (van Dam, 2013 [2005])
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Analogy: Corollary VI
Desirable:

- Show that degree of violation of IC (hence of TB) $\propto$ 'degree of triviality'

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- not an 'appeal to intuition'
- aim to identify the necessary suppositions implicit in any such theories and in our practice of them
- what is required for empirical testing to be possible?
- what distinctions must we make in order to quantify communicational resources?

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- Distinction between distributed and localised computational tasks becomes 'blurry' / 'confused' for the purposes of a complexity-theoretic analysis if IC is violated.
- IC as a physically motivated constraint on mutual independence of two mathematical theories


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